# INDIAN INSTITUTE OF MANAGEMENT KOZHIKODE 

## Term: I (ePGP-03 / eMEP-10) <br> Subject: Operations Research; Code: 112

## End Term Exam.

## Note:

## Attempt any 5 questions out of 6 questions; Each question carries 10 marks.

 Max. Marks: 50; Time: $\mathbf{2}$ hours 30 minutes
## 1) 10 Marks

The Portfolio Manager of MaxWorth Investment Inc. has been asked to invest $\$ 2,000,000$ of a large pension fund. The Investment Research Department has identified six mutual funds with varying investment strategies, resulting in different potential returns and associated risks, as summarized in Table below.

Table: Risk and Expected Rate of Return for Six Mutual Funds

|  | Fund |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 |  |
| Price $(\$ /$ Share $)$ | 55 | 68 | 115 | 18 | 26 | 20 |  |
| Expected return <br> (\%) | 30 | 22 | 15 | 12 | 10 | 8 |  |
| Risk category | High | High | High | Medium. | Medium | Low |  |

One way to control the risk to limit the amount of money invested in the various funds. To that end, the management of MaxWorth Investment Inc. has specified the following guidelines:

- The total amount invested in high-risk funds must be between 50 and $70 \%$ of the portfolio.
- The total amount invested in medium-risk funds must be between 20 and $35 \%$ of the portfolio.
- The total amount invested in low-risk funds must be at least $5 \%$ of the portfolio.

A second to control risk is to diversify-that is, to spread the risk by investing in many different alternatives. The management of MaxWorth Investment Inc. has specified that the amount invested in the high risk Funds 1, 2, and 3 should be in the ratio 1:2:3 respectively. The amount invested in the medium-risk funds 4 and 5 should be 2:1.

With these guidelines, what portfolio should you, the Portfolio Manager, recommend so as to maximize the expected rate of return? Provide LP formulation.
i) List out the assumptions in LP models
ii) Briefly define "binding constraints"

## 2b) 7 Marks

Given this LP model:
Maximize $\quad Z=x_{1}+5 x_{2}$
Subject to

$$
\begin{aligned}
x_{1}+3 x_{2} & \leq 12 \\
3 x_{1}+4 x_{2} & =24 \\
x_{1} & \leq 6 \\
x_{1}, x_{2} & \geq 0
\end{aligned}
$$

- Determine the optimal values of the decision variables using Graphical approach
- Compute the optimal value of the objective function.
- Is any constraint redundant? If so, which one?


## 3) - 10 Marks

The A-B-C department of a large company makes three products ( $A, B$, and $C$ ). The department is preparing for its final run next week, which is just before the annual twoweek vacation during which the entire department shuts down. The manager wants to use up existing stocks of the three raw materials used to fabricate products $A, B$, and $C$. She has formulated the LP model and obtained an optimal solution using Excel, which is displayed in Table attached.

$$
\begin{aligned}
& A=\text { quantity of Product } A \\
& B=\text { quantity of Product } B \\
& C=\text { quantity of Product } C \\
& \text { Maximize } \quad Z=12 A+15 B+14 C \\
& \text { Subject to } \\
& \text { Material } 13 A+5 B+8 C \leq 720 \text { pounds } \\
& \text { Material } 22 A \quad+3 C \leq 600 \text { pounds } \\
& \text { Material } 34 A+6 B+4 C \leq 640 \text { pounds }
\end{aligned}
$$

$$
A, B \text {, and } C \geq 0
$$

As a staff person, the manager has asked you to answer each of the following questions concerning the final solution:
a) Although Product B is the most profitable, and Product A the least profitable, the solution calls for making none of $B$ but 112 of $A$. Why?
b) If $B$ 's profit per unit could be increased to $\$ 18$, how much $B$ would be produced? Explain how you obtained your answer.
c) What is the range of feasibility for the Material 3 RHS?
d) By how much would profit increase if an additional 100 pounds of material 3 could be obtained as its usual cost? What if the amount were an additional 400 pounds?
e) Do you see any difficulty in allowing the A-B-C department to take 200 pounds of material 2? Explain.
f) If the material 1 is increased by 50 pounds, by how much would profit increase?
g) In order to increase profit, which material you will supply more and why?

| Microsoft Excel 11.0 Answer Report <br> Worksheet: [The A-B-C Department.xIs]Model <br> Report Created: 13/2011 6:22:28 PM |  |  |
| :---: | :---: | :---: |
| Target Cell (Max) |  |  |
| Cell Name | Original Value | Final Value |
| \$E\$13 Profit (Max.) | $\bigcirc$ | 2016 |
| Adjustable Cells |  |  |
| Cell Name | Original Value | Final Value |
| \$B\$10 Product A | 0 | 112 |
| \$C\$10 Product B | 0 | 0 |
| \$D\$10 Product C | 0 | 48 |

Constraints

| Cell | Name | Cell Value | Formula | Status | Slack |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\$ E \$ 16$ | Material 1 LHS | $720 \$ E \$ 16<=\$ G \$ 16$ | Binding | 0 |  |
| $\$ E \$ 17$ | Material 2 LHS | $368 \$ E \$ 17<=\$ G \$ 17$ | Not Binding | 232 |  |
| $\$ E \$ 18$ | Material 3 LHS | $640 \$ E \$ 18<=\$ G \$ 18$ | Binding | 0 |  |

Microsoft Excel 11.0 Sensitivity Report
Worksheet: The A-B.C Department.xis]Model
Report Created: 1/3/2011 6:22:28 PM

Adjustable Cells

| Cell | Name | $\begin{array}{c}\text { Final } \\ \text { Value }\end{array}$ |  | $\begin{array}{c}\text { Reduced } \\ \text { Cost }\end{array}$ | $\begin{array}{c}\text { Objective } \\ \text { Coefficient }\end{array}$ | $\begin{array}{c}\text { Allowable } \\ \text { lincrease }\end{array}$ |
| :---: | :---: | ---: | ---: | ---: | ---: | ---: | \(\left.\begin{array}{c}Allowable <br>

Decrease\end{array}\right]\)

Constraints

| Cell | Name | Final <br> Value | Shadow <br> Price | Constraint <br> R.H. Side | Allowable <br> Increase | Allowable <br> Decrease |
| :---: | :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\$ E \$ 16$. Material 1 LHS | 720 | 0.4 | 720 | 560 | 240 |  |
| $\$ E \$ 17$ | Material 2 LHS | 368 | 0 | 600 | $1 E+30$ | 232 |
| $\$ E \$ 18$ | Material 3 LHS | 640 | 2.7 | 640 | 320 | 280 |

4) 10 Marks

Consider the transportation problem of the Home Appliance Manufacturing Company. The demand at each store and the supply available at each of the warehouse are given
in the table. The transportation cost per unit, from warehouse to store is provided in the respective cells.

| From To | Store 1 | Store 2 | Store 3 | Supply |
| :---: | :---: | :---: | :---: | :---: |
|  | 12 | 20 | 15 |  |
| Warehouse A |  |  |  | 50 |
|  | 9 | 11 | 4 |  |
| Warehouse B |  |  |  | 15 |
|  | 20 | 14 | 8 |  |
| Warehouse C |  |  |  | 55 |
| Demand | 25 | 50 | 45 | 120 |

a) Develop an initial feasible solution using the northwest-corner method. Compute the total cost for this solution.
b) Evaluate the solution using the stepping-stone method. Is the solution optimal? Explain.
c) What is the total cost for your optimal solution?

## 3 Marks

Briefly define or explain each of these terms.

- Shadow price.
- Range of optimality.

5b) 7 Marks
A manager has four jobs that must be assigned. Estimated processing times for each employee are shown in the accompanying table.

|  |  | Employee |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Anjan | Abhi | Rishi | Niraj |
|  | 1 | 6.2 | 8.0 | 5.4 | 4.8 |
|  | 2 | 6.0 | 7.2 | 5.8 | 4.4 |
|  | 3 | 5.5 | 6.0 | 6.6 | 6.8 |
|  | 4 | 6.3 | 6.6 | 7.0 | 7.3 |

- Determine a set of assignments that will minimize total processing time.
- What is the total processing time for the optimal assignments?

6) 10 Marks

A salesman has to travel from West to East of India. Although his starting point and destinations were fixed, he has considerable choice as to which territories (or cities) to
travel through on route. The possible routes are shown in the figure where each city is represented by a number block. Thus four 'stages' were required to travel from his embarkation in city 1 to his destination city 10 . The cost of traveling from one city to the other in any feasible route is given in the diagram (cost expressed in some monetary units). The salesman's decision problem is to choose the best possible route from city 1 to 10 so that the overall cost of traveling will be minimum. Use Dynamic Programming approach to solve this problem.


